Corrections to scaling from 'non-dangerous' irrelevant operators

Lets also briefly consider the effect of irrelevant operators on which quantities of physical interest depend in an <u>analytical</u> (i.e. non-singular) way. For example, in the ϕ^4-theory, at the Gaussian fixed point, magnetization depends analytically on the coefficient v of a $v\phi^6$ term. As one may anticipate, such operators do not change the leading scaling behavior of various physical quantities. Consider for example, the dependence of free energy in an Ising model on an irrelevant variable v:

$$f_s = b^{-d} f_s(t\, b^{y_t}, v\, b^{y_v})$$

where $y_v < 0$ and we have set $h=0$ to keep it simple. Again setting $t\, b^{y_t} \sim 1$

$$\Rightarrow \quad f_s = t^{d/y_t} \, \phi(v \, t^{-y_v/y_t})$$

$$= t^{d/y_t} \, \phi(v \, t^{|y_v|/y_t})$$

where ϕ, by assumption is analytic in v.

Therefore we may perform a Taylor expansion of $\phi(x)$ around $x = 0$:

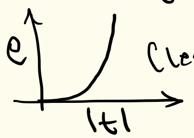
$$\phi(x) = c_0 + c_1 x + c_2 x^2 + \ldots$$

$$\Rightarrow \quad f_s \sim t^{d/y_t} \left(c_0 + c_1 v \, t^{|y_v|/y_t} + \ldots \right)$$

$$\sim c_0 t^{d/y_t} + c_1 v \, t^{(d + |y_v|)/y_t} + \ldots$$

Therefore, very close to the critical point where $|t| \ll 1$, the irrelevant variable only leads to subleading corrections to scaling.

It's still worth noting that the fractional subleading correction $e \sim |t|^{|y_v|/y_t}$ qualitatively behaves differently depending on whether

$|y_v|/y_t > 1$: 

(less singular), or $|y_v|/y_t < 1$: